Problem 1

a) f(1) = ±1, 函数在其定义域中的元素取值不唯一, f(n)不是函数.

b) ∀n(n∈Z→n²+1≥0), 对任意n∈Z存在唯一√(n²+1)∈R, f(n)是从Z到R的函数.

c) 当n=2或n=-2时, n²-4 = 0, f(n) = 1/(n²-4)无意义, f(n)不是从Z到R的函数.

Problem 2

a) f(x)的定义域和值域均为R且在R上单调递减, 有反函数f^-1(x) = (4-x)/3,

则f(x)是从R到R的双射函数.

b) f(1) = f(-1) = 4, 且f(x)值域为(-∞, 7] ≠ R, 则函数f(x)在R上既不是单射也不是满射,

f(x)不是从R到R的双射函数.

c) 当x=-2时x+2=0, f(x)在x=-2处无定义, f(x)有反函数f^-1(x) = (1-2x)/(x-1),

反函数在x=1处无定义, 则f(x)是双射函数但不是从R到R的.

d) f(x)的定义域和值域均为R且在R上单调递增, 有反函数f^-1(x) = (x-1)^1/5,

则f(x)是从R到R的双射函数.

Problem 3

a) {x | x²=1} = {-1, 1}

b) {x | 0<x²<1} = {x | -1<x<0∨0<x<1}

c) {x | x²>4} = {x | x<-2∨x>2}

Problem 4

a≠0, 若x1≠x2, f(x1)-f(x2) = a(x1-x2)≠0, f(x1)≠f(x2).

若f(x1)≠f(x2)即ax1+b≠ax2+b, f(x1)-f(x2) = a(x1-x2)≠0, x1≠x2.

则f(x)既是单射又是满射, f(x)是双射, f(x)可逆, 反函数f^-1(x) = (x-b)/a.

Problem 5

a) f(0, -n) = n, 对-n∈Z有n∈Z, 值域为Z, f: Z×Z→Z是满射的.

b) 取m²-n² = 2即(m+n)(m-n) = 2, m∈Z且n∈Z则(m+n)∈Z, (m-n)∈Z.

2为偶数, 则(m+n)与(m-n)至少有一个为偶数, 假设(m+n)为偶数, (m-n) = (m+n)-2n.

n∈Z, 2n为偶数, 则(m-n)为偶数, (m+n)(m-n)必为4的倍数, 2不是4的倍数, 矛盾.

因此不存在m∈Z, n∈Z使f(m, n) = 2, f: Z×Z→Z不是满射的.

c) f(m, n) = m²-4的值域为[-4, +∞)≠R, f: Z×Z→Z不是满射的.

d) f(0, n) = -|n|, 对n∈Z有-|n|∈(-∞, 0 ],

f(m, 0) = |m|, 对m∈Z有|m|∈[0, +∞), f(m, n)的值域为Z, f: Z×Z→Z是满射的.

e) f(-1, n) = n, 对n∈Z值域为Z, f: Z×Z→Z是满射的.

Problem 6

f◦g = IY则对任意y∈Y, g(y) = x, f(g(y)) = f(x) = y, g^-1 = f.

g◦f = IX则对任意x∈X, f(x) = y, g(f(x)) = g(y) = x, f^-1 = g.

f^-1 = g则对任意x∈X, f(x) = y, f^-1(y) = g(y) = g(f(x)) = x, f◦g = IY.

g^-1 = f则对任意y∈Y, g(y) = x, g^-1(x) = f(x) = f(g(y)) = y, g◦f = IX.

综上所述, f◦g = IY, g◦f = IX与f^-1 = g, g^-1 = f 等价.

Problem 7

若f是单射, 可得|f(A)|=|A|, 又|A|=|B|, |f(A)|=|B|, 则f是满射.

若f是满射, 假设f不是单射, 可得|f(A)|<|A|，|f(A)|<|B|, f不是满射, 矛盾.

则f是单射↔f是满射, f是单射当且仅当它是满射.

Problem 8

a) 1° 取任意的y, 若y∈f(S∪T) , 存在x∈S∪T使得f(x)=y.

若x∈S, 则y∈f(S), 若x∈T, 则y∈f(T), 综上可知y∈f(S)∪f(T), f(S∪T) ⊆f(S)∪f(T)

2° 取任意的y，若y∈f(S)∪f(T)

情况1：y∈f (S), 存在x∈S ⊆S∪T使得f(x)=y, y∈f(S∪T)

情况2：y∉f(S)且y∈f(S)∪f(T), 则y∈f(T), 存在x∈T⊆S∪T使得f(x)=y, y∈f(S∪T)

综上可知y∈f(S∪T), f(S)∪f(T) ⊆f(S∪T), 综上f(S∪T) = f(S)∪f(T).

b) 取任意的y, 若y∈f(S∩T), 存在x∈S∩T使得f(x)=y, x∈S且x∈T,

则y∈f(S)且y∈f(T), b∈f(S)∩f(T), f(S∩T) ⊆f(S)∩f(T).